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New Invariance in Electroweak Standard Model and Conservation of Helicity

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The Lagrangian of the electroweak standard model is invariant under the chirality transformation of fermionic fields and reversal of the Higgs field. This invariance implies the conservation of helicity in weak interaction processes. The application to leptonic weak interaction is discussed in detail.

In the electroweak standard model (Glashow, 1961; Weinberg, 1967; Salam, 1968), two vacuums are situated in a symmetric position, and the Lagrangian is invariant under the chirality transformation of fermionic fields, the reversal of the vacuum expectation value v, and the reversal of the Higgs field $\eta(x)$. Suppose $\psi(x)$ denotes the lepton fields ($l=e, \mu, \tau$) and quark fields ($q=u, d, s, c$). Such a transformation is

$$\psi(x) \to \psi'(x) = \gamma_5 \psi(x) \tag{1a}$$

$$\bar{\psi}(x) \to \bar{\psi}'(x) = -\bar{\psi}(x)\gamma_5 \tag{1b}$$

$$\eta(x) \to \eta'(x) = -\eta(x) \tag{1c}$$

$$v \to v' = -v \tag{1d}$$

and other fields do not change.

In the electroweak standard model, the masses of leptons and quarks are given by

$$m_l = \frac{1}{\sqrt{2}} G_l v, \qquad m_q = \frac{1}{\sqrt{2}} G_q v \tag{2}$$

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respectively. The transformation (1) is equivalent to the mass reversal of the spinor field $\psi(x)$. There is a unitary operator M for the spinor field such that

$$M\psi(x, m)M^{-1} = \gamma_5\psi(x, -m) \tag{3a}$$

$$M\bar{\psi}(x, m)M^{-1} = -\bar{\psi}(x, -m)\gamma_5 \tag{3b}$$

Expanding the spinor field

$$\psi(x, m) = V^{-1/2}\sum_k \sum_{r=1}^{2} [a_r(k)u_r(k, m)\, e^{ikx} + b_r^+(k)v_r(k, m)\, e^{-ikx}] \tag{4a}$$

$$\bar{\psi}(x, m) = V^{-1/2}\sum_k \sum_{r=1}^{2} [a_r^+(k)\bar{u}_r(k, m)\, e^{-ikx} + b_r(k)\bar{v}_r(k, m)\, e^{ikx}] \tag{4b}$$

where

$$\begin{aligned} u_r(k, m) &= \left(\frac{E+m}{2E}\right)^{1/2} \begin{pmatrix} \xi_r \\ (\boldsymbol{\sigma}\cdot\mathbf{k})/(E+m)\xi_r \end{pmatrix}, \\ v_r(k, m) &= \left(\frac{E+m}{2E}\right)^{1/2} \begin{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{k})/(E+m)\xi_r \\ \xi_r \end{pmatrix} \end{aligned} \tag{5}$$

The ξ_r are Pauli spinors, which may be chosen as eigenstates of the helicity. Obviously,

$$u_r(k, -m) = \frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{(\mathbf{k}^2)^{1/2}}\, v_r(k, m) \tag{6a}$$

$$v_r(k, -m) = \frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{(\mathbf{k}^2)^{1/2}}\, u_r(k, m) \tag{6b}$$

Suppose the operators $a(k)$, $b(k)$, $a^+(k)$, and $b^+(k)$ are independent of the sign of the parameter m (Ouch *et al.*, 1956). Then

$$\psi(x, -m) = V^{-1/2}\sum_k \sum_{r=1}^{2} [a_r(k)u_r(k, -m)\, e^{ikx} + b_r^+(k)v_r(k, -m)\, e^{-kx}] \tag{7a}$$

$$\bar{\psi}(x, -m) = V^{-1/2}\sum_k \sum_{r=1}^{2} [a_r^+(k)\bar{u}_r(k, -m)\, e^{-ikx} + b_r(k)\bar{v}_r(k, -m)\, e^{ikx}] \tag{7b}$$

Substitute (4a) and (7a) into (3a) and substitute (4b) and (7b) into (3b); one obtains

$$Ma_r^+(k)M^{-1} = -ha_r^+(k) \tag{8a}$$

$$Mb_r^+(k)M^{-1} = -h'b_r^+(k) \tag{8b}$$

etc., where h is an eigenvalue of the helicity operator.

Consider a weak interaction process that in initial and final states contains only fermions. Suppose $|0\rangle$ denotes the vacuum state. The state $|n, n'\rangle$ of n fermions and n' antifermions may be written as

$$|n, n'\rangle = \sum_{k,l,m} R_l(\mathbf{k}, r) Y_l^m(\theta, \varphi)\sigma(s) a_1^+ \ldots a_n^+ b_1^+ \ldots b_{n'}^+ |0\rangle \tag{9}$$

Suppose that (Ouch *et al.*, 1956)

$$M|0\rangle = |0\rangle \tag{10}$$

Using (8a) and (8b), one obtains

$$\begin{aligned} M|n, n'\rangle &= \sum_{k,l,m} R_l(\mathbf{k}, r) Y_l^m(\theta, \varphi)\sigma(s) M a_1^+ M^{-1} \ldots M a_n^+ M^{-1} M b_1^+ M^{-1} \\ &\qquad \ldots M b_{n'}^+ M^{-1} M |0\rangle \\ &= \mathscr{H} |n, n'\rangle \end{aligned} \tag{11}$$

where

$$\mathscr{H} = (-1)^{n+n'} h_1 \ldots h_n h_1' \ldots h_{n'}' \tag{12}$$

h_i are the helicities of the fermions and h_i' are the helicities of the antifermions. The Hamiltonian of the electroweak standard model is invariant under the transformation (1); then M is a conservation operator, and the eigenvalue $\mathscr{H}$ is a conserved quantity of the system.

In application to weak interaction processes of leptons, for example, $\mu^{\pm}$ decay, the helicities of the neutrino and antineutrino are $h(\nu) = -1$ and $h(\tilde{\nu}) = 1$, respectively. From the $e^{\pm}$ helicities in muon decay (Commins and Bucksbaum, 1983), according to the conservation law of the helicity (12), we can conclude that $h(\mu^{\pm}) = \mp 1$, which is in agreement with experiment and consistent with conservation of the momentum and angular momentum in the decay $\pi \to \mu\nu$. Suppose the decay of the τ leptons is similar to muon decay. We can then determine the helicity of the τ leptons. For the neutrino–electron elastic scattering, reactions $\nu_e e^- \to \nu_e e^-$, $\tilde{\nu}_e e^- \to \tilde{\nu}_e e^-$ involve both charged and neutral currents, and the experiments give results in good agreement with the standard model; the conservation law (12) implies that the helicity of the electron does not change in this scattering. Similarly, in the processes $\nu_\mu e^- \to \nu_\mu e^-$, $\tilde{\nu}_\mu e^- \to \tilde{\nu}_\mu e^-$, $\nu_\mu e^- \to \mu^- \nu_e$ leptons have the same helicity before and after scattering. Applied to the reaction $e^- e^+ \to l^- l^+$, which involves weak electromagnetic interference, (12) relates the helicities of the leptons in these processes.

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